1. [Start of transcript. Skip to the end.](https://courses.edx.org/xblock/block-v1:ColumbiaX+CSMM.101x+3T2020+type@vertical+block@d1cc78c3b86e4418a8ce97f46e1c551b?show_title=0&show_bookmark_button=0#transcript-end-936e34e904ee4c35881855b1a61eb3a6)
2. Among the connectives, the implication
3. plays a crucial role in inference.
4. Let's see first it's different variants
5. and also clarify the terminology.
6. So first of all, an implication is a Boolean expression,
7. or actually, a proposition of the form p implies q.
8. So sometimes you would see it written as p
9. implies q, but this is the same.
10. OK, so we call, also, converse, the implication
11. that goes from the right hand side to the left hand side,
12. q implies p.
13. And also the contrapositive is the negation of the consequent,
14. or the right-hand side, not q implies not p.
15. We call the inverse, if you take the negation of the first side,
16. of left-hand side implies the negation
17. of the right-hand side.
18. It's important to observe here that some of these
19. are equivalent and some are not.
20. So for example, we have seen the previous slides
21. that we could write actually p implies q, is logically
22. equivalent to not q implies not p.
23. So we can prove this easily by doing some truth table.
24. So we could do that quickly here.
25. So we have p implies q, not q implies not p.
26. So if you know the truth table to prove that these two
27. expressions are logically equivalent,
28. you're going to put the possible values for p and q truth
29. values, so false-true and false-false.
30. So just to iterate on the truth value of the implication,
31. the implication is false only when
32. we have true implies false.
33. So it's going to be here false.
34. And for the rest, it's true.
35. OK, so not q implies not p would be not q, where
36. you are going to negate this element here and this element
37. here, not q implies not p.
38. This is false implies anything, so that's true.
39. And here we have false implies anything that's true.
40. And here we have true implies true, because we have not
41. q implies not p.
42. So it's going to be true implies true is true.
43. And for the last element here we have not q,
44. it would be true implies false, is false.
45. So you could observe from this truth table
46. that these two columns are actually identical,
47. which means that if you have an implication, p implies q,
48. you could easily replace it by something logically equivalent,
49. which is not q implies not p.
50. So just to clarify these two elements will be,
51. then, logically equivalent.
52. But you could also show that q implies p, and not
53. p implies not q.
54. And also the, actually, logically equivalent,
55. actually the inverse would be the contrapositive
56. of the converse.
57. So q implies not p.
58. It's enough to negate the right-hand side
59. and implies the negation of the left-hand side.
60. So let's take an example.
61. Hot is a sufficient condition for my going
62. to the beach can be written as an implication.
63. If it's hot, then I go to the beach.
64. The converse of this proposition is q
65. implies p, which means that I'm going
66. to take the right-hand side and put it on the left-hand side.
67. And again, I put the left-hand side on the right-hand side,
68. beach implies hot.
69. The contrapositive of this implication
70. here is, which is equivalent to hot implies beach would
71. be not beach implies not hot, while the inverse, which
72. would be the contrapositive of the converse,
73. would be actually not hot implies not beach.
74. And again these two elements are actually logically equivalent.
75. All right so we see how we could use
76. this definition of implication of contrapositive
77. to actually do inference based on some inference rules.
78. Our first inference rule is called modus ponens,
79. which shows how to infer or how to derive logically
80. new sentences from existing ones.
81. So for example, if we have p is true,
82. when we write p by itself, which means that if p is true,
83. and if p plus q is true, then we can infer or make
84. a logical deduction that actually q is true.
85. So this one here stands for inference.
86. This is an inference with the modus ponens, MP.
87. All right, so if I observe p and p is true, and if I have p
88. implies q is true, then I can confidently
89. infer that q is also true.
90. So how can you write this actually
91. as a Boolean expression, because here we are not
92. used to write this line of inference,
93. but in fact, you can find it easily as an implication.
94. So this means that there is an implication this way.
95. So we could write this Boolean expression as p, if I have p,
96. and I observe p implies q.
97. So this means that if these two elements are true, then
98. I can confidently imply that actually Q is true.
99. As a matter of fact, if you write the truth table for p, q,
100. and this expression here, so if you do the truth table,
101. you will find that it is actually a tautology.
102. So it's always true that if you have p true,
103. and you have p implies q true, the conjunction would
104. imply that q is also true.
105. As an example, if we have the proposition warm
106. and the other proposition warm implies sunny,
107. then I can confidently infer that it is sunny.
108. So warm is true.
109. And warm plus sunny is true, then sunny is true.
110. And this is coming from simply the definition
111. of implication itself.
112. We let the implication warm implies sunny is actually
113. cannot be true if the left-hand side is true and the right-hand
114. side is false.
115. So the only way where this implication is false
116. and when we have true implies false.
117. Given that this part is true, and the whole implication
118. is true, it means that we must have that sunny is also true.
119. So this is a very powerful inference mechanism
120. that we can use and reuse when we have a knowledge base
121. to derive new sentences and make inference about the work.
122. Modus Ponens can be extended by handling
123. what we call Horn clauses It's a proposition of the form p1,
124. and p2, and pn implies q.
125. So now on the left-hand side we don't
126. have only one single proposition,
127. we had a conjunction of propositions.
128. Modus ponens deals with Horn clauses is as follows.
129. So we're going to just extend it.
130. Instead of having just p and p implies q,
131. now we have if p1 is true, p2 is true, pn is true,
132. and if we have p1, and p2, and pn implies q,
133. then we can confidently infer q.
134. So this is just a way to say it in a marginal fashion
135. that if all these elements are true, then
136. we know that their conjunction is true.
137. If the implication is true, then we
138. can infer that q is also true.
139. Another variant of modus ponens is called the modus tollens.
140. So in modus tollens, we're going to leverage the fact
141. that an implication, p implies q,
142. is logically equivalent to its contrapositive,
143. not q implies not p.
144. So this is the key idea of modus tollens.
145. So we have p implies q is equivalent to not q implies not
146. p.
147. So if we have this, then if we have p implies q,
148. and we observe not q, so look at the rules.
149. So if we observe not q, by modus tollens, which
150. is actually applying this contrapositive,
151. we can infer not p.
152. So not p and p implies q would infer not p.
153. So this is just so we could rewrite
154. that is not q implies not p.
155. You could, if you were to rewrite this one,
156. then you could see that by modus ponens, not q
157. and not q implies not p would need to not p.
158. But if we rewrite just as p implies q,
159. we call this modus tollens.
160. This is pretty powerful as well.
161. So in case we want to infer not the negation of a proposition,
162. we could use modus tollens as an inference tool.
163. As an example, if we consider again the example,
164. hot implies beach.
165. So probably this is, this implication could be used along
166. with hot, if I observe that it's hot,
167. I could infer for sure that I am going to go to the beach.
168. This is what I do always when it's hot.
169. Now if I want to use it in another way,
170. hot implies beach, which is not beach implies not hot.
171. So I'm going to use the fact that if it's not,
172. if I don't go to the beach, then it means that it's not hot.
173. And if I observe that I didn't go to the beach,
174. then I could infer that it's not hot.
175. So you could say that we could do inference with different,
176. with these two aspects of the implication, which
177. is pretty powerful.
178. This one is called modus tollens.
179. And this one is by modus ponens.
180. When we do inference, we can also
181. use a set of all the common rules.
182. And this included what you call addition.
183. Addition means that you are using
184. the connective of disjunction.
185. So if we have two propositions, p and q,
186. so the addition means that if p is true,
187. then you can add to it anything.
188. If p is true, then by definition of the disjunction,
189. p or anything else is also true, because one of them
190. must be true.
191. So p infers p or q.
192. The rule of simplification is by removing
193. one of the propositions from a conjunction.
194. So if we know by definition of the conjunction
195. that p and q is true, then any of these elements
196. of the proposition in the conjunction is also true.
197. So if p and q is true, then you could
198. infer that q is also true.
199. And you could also have p and q is true.
200. You could also infer that p is true.
201. So a conjunction of literals can actually
202. infer the truthfulness of any of the literals in the formula.
203. We talk disjunctive-syllogism.
204. This is or q and not p.
205. So if p of q is true, and we have not p is true,
206. mean that p is false, then p or q is actually false,
207. or q, which will actually, we leave it up to q.
208. So if the truth value of this expression would be simply q.
209. So if p or q is true, then one of them must be true.
210. And we know that p is false, then it would depend on q.
211. So it could simplify these two elements here, and infer
212. that it's would depend on q.
213. And finally what we call a hypothetical-syllogism is
214. syllogism in which we have if p implies q, and q implies r,
215. it's kind of transitivity.
216. Given that these rules are actually 100 percent sure,
217. then we have p plus q is true.
218. Q implies r is true.
219. Then we must have that p implies r is also true.
220. So these are all useful rules when
221. we do inference to derive new sentences from the launch base,
222. to make actually new sentences.
223. This table now summarizes the different connectives.
224. So we have p and q atomic propositions
225. on the left-hand side, separated by double lines
226. with a set of compound propositions.
227. So we have the negation of the proposition, the conjunction,
228. the disjunction, the implication,
229. and double implication, or if and only if.
230. All right, so we have a truth table of size 2
231. to the 2, which is the number of propositions, which would be
232. four lines in the truth table.
233. Just to clarify the terminology of the implications,
234. so we have p implies q is the implication.
235. So we need to sometimes refer to the left-hand side
236. and the right-hand side.
237. So we call this left-hand side, this part right-hand side.
238. Sometimes you are also here talking
239. about the premise and the conclusion,
240. or also the body, and the head, or condition conclusion,
241. and conclusion.
242. This is different terms to refer to the left-hand side
243. and the right-hand side of the implication.
244. So we see later how we use truth tables to make proofs.
245. [End of transcript. Skip to the start.](https://courses.edx.org/xblock/block-v1:ColumbiaX+CSMM.101x+3T2020+type@vertical+block@d1cc78c3b86e4418a8ce97f46e1c551b?show_title=0&show_bookmark_button=0#transcript-start-936e34e904ee4c35881855b1a61eb3a6)